Rates of Change involving KINEMATICS

Kinematics = study of motion along a straight-line path. The relationship between distance, velocity, and acceleration is displayed below.

$$\text{Distance (s)} \underset{s = \int v\,dt}{\overset{v = \frac{ds}{dt}}{\rightleftarrows}} \text{Velocity (v)} \underset{v = \int a\,dt}{\overset{a = \frac{dv}{dt}}{\rightleftarrows}} \text{Acceleration (a)}$$

Conventions

Several conventions hold in kinematics problems.

1) **Initial values** apply when time is 0, i.e. when motion started.

2) **Zero values**
  a) Distance = 0 means object is at the **origin**
  b) Velocity = 0 means the object is **stationary**.
  c) Acceleration = 0 means the object is travelling at **constant speed**.

3) **Positive values**
  a) Distance > 0 means object is **above** or to the **right** of the origin.
  b) Velocity > 0 means object is travelling **forwards** or **upwards**.
  c) Acceleration > 0 means object is **speeding up**.

4) **Negative values**
  a) Distance < 0 means object is **below** or to the **left** of the origin.
  b) Velocity < 0 means object is travelling **backwards** or **downwards**.
  c) Acceleration < 0 means object is **slowing down**.

5) $\int_{t_1}^{t_2} v(t)\,dt$ gives the distance travelled in between t_1 and t_2 seconds.

Examples

1. A stone is thrown vertically upwards into the air. Its height in metres above the ground is given by $s(t) = 30t - 5t^2$.

   a) Calculate the initial velocity
   b) Calculate the maximum height reached by the object
   c) When does the object return to the ground?

a) Velocity $v(t) = s'(t) = 30 - 10t$
Initial velocity $v(0) = 30 - 10(0) = 30$ m/s

b) Max height occurs when object stops travelling upwards and starts to fall, and this is where its velocity is 0, i.e. solve $v(t) = 0$: $=>$ $30 - 10t = 0$ $=>$ $t = 3$ secs
Therefore, maximum height is reached at $t = 3$ secs:
$s(3) = 30(3) - 5(3)^2 = 45$ metres.

c) Object reaches ground when it's distance from the ground is 0, so solve $s(t) = 0$:
$30t - 5t^2 = 0$ $=>$ $5t(6 - t) = 0$ $=>$ $t = 0$ and $t = 6$.
Therefore the object returns to the ground at $t = 6$ seconds.

2. The velocity (in m/s) of an object t seconds after it started from the origin is given by $v(t) = 3t^2 - 14t - 5$. The object is travelling in a straight line in an East-West direction.

a) Give the formulae for the acceleration and distance after t seconds.
b) Calculate the initial velocity. In which direction is the object moving?
c) When is the object at rest?
d) Calculate the minimum velocity. Interpret this answer.
e) How far did the object travel in the eighth second?

a) Distance, $s = \int v\, dt = t^3 - 7t^2 - 5t + c$.
Object started from origin, so $s = 0$ when $t = 0$, giving $c = 0$.
Therefore $s = t^3 - 7t^2 - 5t$

Acceleration, $a = \frac{dv}{dt} = 6t - 14$.

b) Initial velocity, $v(0) = 3(0)^2 - 14(0) - 5 = -5$ m/s. Object is travelling East.

c) Object is at rest when $v(t) = 0$:
$3t^2 - 14t - 5 = 0$ $=>$ $(3t + 1)(t - 5) = 0$ $=>$ $t = -\frac{1}{3},\ t = 5$

d) Minimum velocity occurs when its derivative is 0 i.e. when $a(t) = 0$:
$6t - 14 = 0$ $=>$ $t = 2\frac{1}{3}$ secs.
Min velocity, $v\left(2\frac{1}{3}\right) = 3(2\frac{1}{3})^2 - 14\left(2\frac{1}{3}\right) - 5 = -21\frac{1}{3}$ m/s occurs when object is travelling East.

e) Distance travelled in the eighth second, $\int_7^8 v\, dt = \int_7^8 3t^2 - 14t - 5\ dt$
$= [\frac{3t^3}{3} - \frac{14t^2}{2} - 5t]_7^8 = 59$ metres.

Delta Ex 11.4 pg 130 - 131